Indian J. Phys. **74B** (3), 217–224 (2000)

I J P B
an international journal

Electrostatic instabilities in multi-ion plasmas

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Received 12 June 1998, accepted 18 January 2000

Abstract : We have studied the stability of the ion acoustic wave in a multi-ion plasma consisting of a lighter ion component (hydrogen) and positively and negatively ionised heavier ion components of oxygen. This model approximates very well the plasma environment of the coma of comet Halley.

Coupled equations have been derived for the dispersion relation as well as the growth/damping rate for oblique propagation of ion-acoustic waves, these equations were solved by numerical methods. Various aspects of wave stability with regard to angle of propagation, number densities and species temperatures have been considered. While for all combinations of compositions the wave growth is a maximum for parallel propagation, the heavier species (irrespective of its charge) either aids/damps the instability depending on whether they are hotter/colder than the lighter hydrogen ions.

Keywords : Multi-ion plasmas, Halley's comet

PACS Nos. : 52.35.Qz, 96.50.Gn

1. Introduction

The frequent observations of low frequency wave activity in space plasma environments climaxed with satellite missions to the comets Halley and Giacobini-Ziner. These cometary missions spurred intense research efforts which were aimed at understanding the corresponding excitation mechanisms and free energy sources. Theoretical stability analyses have considered background hydrogen magnetoplasmas permeated by proton (or heavier) ion beams with Maxwellian [1] or drifting ring [2–4] velocity distributions and parallel propagation. Oblique propagation of these waves have also been considered in a number of instances [5–8].

Many of the above studies either modelled cometary environments or were easily adaptable to them. They did not, however, take into consideration the co-existence of more than one new-born ion species and their eventual interaction. The consideration of the new-born ions fuelled yet another burst of intense theoretical activity aimed at understanding the characteristics of hydromagnetic [9,10] and electromagnetic [11,12] waves associated with these ions. These studies generally modelled the magnetoplasma with two (oxygen and hydrogen) ions; the effect of water ions were also considered.

Wave stability has thus been well investigated in cometary plasma environments containing more than one species of positive ion. However, cometary plasmas also contain negative ions in abundant number, especially the coma of comet Halley [13]. Instability studies of waves propagating in such environments have unfortunately neglected this aspect of the composition of the plasma, though such investigations have been attracting some attention of late [14,15].

The plasma wave probe carried by the spacecraft Sagigake detected a number of electrostatic and electromagnetic waves in the environment of comet Halley. Among the electrostatic waves, electron plasma and ion-acoustic waves were clearly detected. The ion-acoustic waves had a frequency range of a few hundred KHz [16]. Again, the plasma wave instrument on the International Cometary Explorer (ICE) spacecraft detected, in addition to plasma waves, strong bursts of ion-acoustic waves when the spacecraft was within 2 million km of the comet Giacobini-Ziner [17]. Ion-acoustic waves are thus an important mode generated in the plasma environment of a comet.

We have considered the stability of the ion-acoustic wave in a plasma having positive ions of hydrogen (H) and singly ionised oxygen (O^+) and also negative ions, again of oxygen (O^-). Such a composition approximates the plasma of the coma of comet Halley very well; though more than one species of negative ions have been found [13].

Various aspects of wave instability with regard to angle of propagation, number densities, species temperatures, *etc.* have been considered. While for all compositions we find that wave growth is a maximum for parallel propagation, the heavier species (irrespective of its charge) either aids/damps the instability depending on whether they are hotter/colder than the lighter species hydrogen.

2. The dispersion relation

We are interested, in this paper, on the stability of the ion-acoustic wave in a plasma containing hydrogen (H) and singly ionised oxygen (O^+) as the positively charged species. The electrons and the negatively charged oxygen (O^-) provide charge neutralisation. Also the hydrogen ions drift with a velocity U with respect to the two heavier species O^+ and O^-. Under these conditions, the dispersion relation for electrostatic waves in a homogenous, unmagnetised plasma can easily be written by extending earlier relations for a single ion plasma [18] as

$$1 = \frac{\omega_{pe}^2}{k^2\alpha_e^2} Z'\left(\frac{\omega}{k\alpha_e}\right) + \frac{\omega_{pH}^2}{k^2\alpha_H^2} Z'\left(\frac{\omega - k_{\parallel}U}{k\alpha_H}\right) + \frac{\omega_{pO^+}^2}{k^2\alpha_{O^+}^2} Z'\left(\frac{\omega}{k\alpha_{O^+}}\right) + \frac{\omega_{pO^-}^2}{k^2\alpha_{O^-}^2} Z'\left(\frac{\omega}{k\alpha_{O^-}}\right) \quad (1)$$

where Z is the plasma dispersion function of Fried and Conte [19], $k_{\parallel}$, the projection of the wave vector $\boldsymbol{k}$ along the drift velocity $\boldsymbol{U}$ and α_j are the thermal velocities, j = e, H, O^+ or O^-. Also ω_{pj} are the plasma frequencies of the different species.

Assuming $\omega = \omega_r + i\gamma$, with $\omega_r >> \gamma$, a Taylor series expansion of the plasma dispersion function, to the first order in γ, yields :

$$Z\left(\frac{\omega}{k\alpha}\right) = Z\left(\frac{\omega_r}{k\alpha}\right) + \frac{i\gamma}{k\alpha} Z'\left(\frac{\omega_r}{k\alpha}\right). \quad (2)$$

We assume $\frac{\omega}{k\alpha_e} << 1$ and $\frac{\quad}{k\alpha_H} >> 1$ [18] and use the Taylor series expansion of Z for the electron background and the asymptotic expansion for the drifting hydrogen ions. For an extensive study of the effect of the heavier ions, we consider both the series and asymptotic expansions of the plasma dispersion functions of the O^+ and O^- ions.

2.1. Case (i) Hot O^+ and O^- ions :

In this section, we derive expressions for the real part of the frequency and the growth/damping rate from the dispersion relation (1), when both types of oxygen ions are hot. As stated above we use the series and asymptotic expansions of Z respectively for the electrons and hydrogen ions. In addition, since the oxygen ions are assumed hot, we need the series expansion of Z for these ions. Substituting these expansions into (1), and carrying out the simplification as indicated in [18] we get for the real part

$$Y^2 = -2\left[\frac{U\cos\theta}{\alpha_e}\right]^2 + \frac{\delta_H \eta_H}{\left[1-\frac{X}{Y}\right]^2} + \frac{3}{2}\left[\frac{\alpha_H}{U\cos\theta}\right]^2 \frac{1}{\left[1-\frac{X}{Y}\right]^2}$$

$$-2\sum_{j=O^+,O^-} \eta_j \delta_j \left[\frac{U\cos\theta}{\alpha_j}\right]^2 \left\{1 - 2\left[\frac{X}{Y}\frac{U\cos\theta}{\alpha_j}\right]\right.$$

$$+ 2\sqrt{\pi}\delta_H \eta_H \frac{\gamma}{Y\omega_{pe}}\left[\frac{U\cos\theta}{\alpha_H}\right]^3 \left\{1 - 2\left(\frac{U\cos\theta}{\alpha_H}\right)^2\left(\frac{X}{Y}-1\right)^2\right\}$$

$$\times \exp\left[-\left(\frac{U\cos\theta}{\alpha_H}\left(\frac{X}{Y}-1\right)\right)^2\right] + 2\sqrt{\pi}\frac{\gamma}{Y\omega_{pe}}$$

$$\times\sum_{j=O^+,O^-} \delta_j \eta_j \left[\frac{U\cos\theta}{\alpha_j}\right]^3$$

$$1 - 2\left(\frac{U\cos\theta}{\alpha_j}\right)^2\left(\frac{X}{Y}\right. \quad \exp\left.-\left(\frac{U\cos\theta}{\alpha_j}\frac{X}{Y}\right.\right. \quad (3)$$

where $X = \frac{\omega_r}{\omega_{pe}}$, $Y = \frac{k_{\parallel}U}{\omega_{pe}}$, $\cos\theta = k_{\parallel}/k$,

$\delta_H = m_e / m_H$, $\delta_{O^+} = m_e / m_{O^+}$, $\delta_{O^-} = m_e / m_{O^-}$,

$\eta_H = \frac{n_H}{n_e}$, $\eta_O^+ = \frac{n_O}{n_e}$, $\eta_{O^-} = \frac{n_O}{n_e}$

with m, indicating the mass and n, the number density; the subscripts indicating the species. The expression for the growth/damping rate can be written as

$$\frac{\quad}{\omega_{pe}} \quad \frac{A}{B} \quad (4)$$

$$A = \sqrt{\pi}x\left[\frac{U\cos\theta}{\alpha_e}\right]^3 \exp\left[-\left(\frac{X}{Y}\frac{U\cos\theta}{\alpha_e}\right)\right.$$

$$-\left(\frac{\alpha_e}{\alpha_H}\right)^3 \delta_H \eta_H \left(\frac{Y}{X}-1\right.$$

$$\left.\times\exp -\left(\frac{U\cos\theta}{\alpha_H}\left(1-\frac{X}{Y}\right)\right)^2\right] + \sum_{j=O^+,O^-}\left(\frac{\alpha_e}{\alpha_j}\right)^3 \delta_j \eta_j$$

$$\times \exp -\left(\frac{U\cos\theta}{\alpha_j}\frac{X}{Y}\right. \quad (5a)$$

and $$B = \frac{\delta_H \eta_H}{\quad} \quad \left(\frac{\alpha_H}{U\cos\theta\left(\frac{X}{Y}-1\right)}\right)$$

$$4\frac{X}{Y}\sum_{j=O^+,O^-}\delta_j\eta_j \times\left(\frac{U\cos\theta}{\alpha_j}\right)^4\left[1 - \frac{4}{3}\left(\frac{X}{Y}\frac{U\cos\theta}{\alpha_j}\right)^2\right]. \quad (5b)$$

The wave grows/damps accordingly as $\frac{\gamma}{\omega_{pe}}$ is positive or negative.

Inspecting (5a), we find that the numerator A is always positive if $Y/X < 1$; or in other-words, $\frac{\omega_r}{k} (= V_{\text{phase}}) > U \cos\theta$ in agreement with earlier results [18]. On the other hand, we find, from the expression (5b) for B that once this condition is met, the contribution of the hydrogen term is always negative. However, though $X/Y > 1$, the contributions from the oxygen ions can be made positive if these ions are hot, in agreement with our assumptions. Thus, the hot, heavier ions can be a source of instability for the ion-acoustic wave.

2.2. *Case (ii) Cold O^+ and O^- ions :*

We next derive expressions for Y^2 and $\frac{\gamma}{\omega_{pe}}$ when the O^+ and O^- ions are cold. We need the series expansion of Z for electrons and the asymptotic expansion of Z for H, O^+ and O^- ions. Proceeding as in Section 2.1, we have

$$Y^2 = -2\left[\frac{U\cos\theta}{\alpha_e}\right]^2 + \frac{\delta_H\eta_H}{\left(1-\frac{X}{Y}\right)^2}\left\{1+\frac{3}{2}\left(\frac{\alpha_H}{U\cos\theta\left(1-\frac{X}{Y}\right)}\right)^2\right\}$$

$$+\frac{1}{\left(\frac{X}{Y}\right)^2}\sum_{j=O^+,O^-}\delta_j\eta_j\left\{1+\frac{3}{2}\left(\frac{\alpha_j}{U\cos\theta}\right)^2\frac{1}{\left(\frac{X}{Y}\right)^2}\right\}$$

$$+2\sqrt{\pi}\delta_H\eta_H\frac{\gamma}{Y\omega_{pe}}\left(\frac{U\cos\theta}{\alpha_H}\right)^3\left\{1-2\left(\frac{U\cos\theta}{\alpha_H}\right)^2\left(\frac{X}{Y}-1\right)^2\right.$$

$$\times\exp\left[-\left(\frac{U\cos\theta}{\alpha_H}\left(\frac{X}{Y}-1\right)\right)^2\right]+2\sqrt{\pi}\frac{\gamma}{Y\omega_{pe}}\sum_{j=O^+,O^-}\delta_j\eta_j$$

$$\times\left(\frac{U\cos\theta}{\alpha_j}\right)^3\left[1-2\left(\frac{U\cos\theta}{\alpha_j}\frac{X}{Y}\right)^2\right]\exp\left[-\left(\frac{U\cos\theta}{\alpha_j}\frac{X}{Y}\right)^2\right] \quad (6)$$

and

$$\frac{\gamma}{\omega_{pe}} = \frac{A}{B} \quad (7)$$

with

$$A = \sqrt{\pi}x\left[\frac{U\cos\theta}{\alpha_e}\right]^3$$

$$\exp\left[-\left(\frac{U\cos\theta}{\alpha_e}\frac{X}{Y}\right)^2\right]-\left(\frac{\alpha_e}{\alpha_H}\right)^3\delta_H\eta_H$$

$$\times\left(\frac{Y}{X}-1\right)\exp\left[-\left(\frac{U\cos\theta}{\alpha_H}\left(1-\frac{X}{Y}\right)\right)^2\right]+\sum_{j=O^+,O^-}\left(\frac{\alpha_e}{\alpha_j}\right)^3$$

$$\times\delta_j\eta_j\exp\left[-\left(\frac{U\cos\theta}{\alpha_j}\frac{X}{Y}\right)^2\right] \quad (8a)$$

and

$$B = \frac{\delta_H\eta_H}{\left(1-\frac{X}{Y}\right)^3}\left[1+3\left(\frac{\alpha_H}{U\cos\theta}\frac{1}{\left(\frac{X}{Y}-1\right)}\right)^2\right]$$

$$-\frac{1}{\left(\frac{X}{Y}\right)^3}\sum_{j=O^+,O^-}\delta_j\eta_j\times\left[1+3\left(\frac{\alpha_j}{U\cos\theta\left(\frac{X}{Y}\right)}\right)^2\right]. \quad (8b)$$

The numerator (8a) is positive if $(Y/X) < 1$ or $V_{\text{phase}} > U\cos\theta$. However, now, the wave can be fully damped since O^+ and O^- also contribute to wave damping.

2.3. *Case (iii) Hot O^+ and cold O^- ions :*

We had in Sections 2.1 and 2.2, derived expressions for the real part of the frequency and the growth/damping rate when the O^+ and O^- ions were either hot or cold. In this section, we consider the case when the O^+ ions are hot and the O^- ions are cold, the other case will not be considered as they are expected to yield similar results. Thus, in addition to the usual expansions of Z for electrons and hydrogen ions, we need the power series expansion of Z for the O^+ ions and the asymptotic expansion for O^- ions. On final simplification, we have

$$Y^2 = -2\left(\frac{U\cos\theta}{\alpha_e}\right)^2+\frac{\delta_H\eta_H}{\left(1-\frac{X}{Y}\right)^2}\left\{1+\frac{3}{2}\left(\frac{\alpha_H}{U\cos\theta\left(1-\frac{X}{Y}\right)}\right)^2\right\}$$

$$-2\delta_{O^+}\eta_{O^+}\left(\frac{U\cos\theta}{\alpha_{O^+}}\right)^2\left[1-2\frac{X}{Y}\frac{U\cos\theta}{\alpha_0^+}\right]$$

$$\times\frac{\delta_O\eta_O}{\left(\frac{X}{Y}\right)^2}\left\{1+\frac{3}{2}\left(\frac{\alpha_O}{U\cos\theta}\right)\frac{1}{\left(\frac{X}{Y}\right)^2}\right\}+2\sqrt{\pi}\delta_H\eta_H\frac{\gamma}{Y\omega_{pe}}$$

$$\times\left(\frac{U\cos\theta}{\alpha_H}\right)^3\left\{1-2\left(\frac{U\cos\theta}{\alpha_H}\right)^2\left(\frac{X}{Y}-1\right)^2\right.$$

$$\times\exp\left[-\left(\frac{U\cos\theta}{\alpha_H}\left(\frac{X}{Y}-1\right)\right)^2\right]+2\sqrt{\pi}\frac{\gamma}{Y\omega_{pe}}\sum_{j=O^+,O^-}\delta_j\eta_j$$

$$\left(\frac{U\cos\theta}{\alpha_j}\right)^3\left[1-2\left(\frac{U\cos\theta}{\alpha_j}\frac{X}{Y}\right)^2\right]\exp\left[-\left(\frac{U\cos\theta}{\alpha_j}\frac{X}{Y}\right)^2\right] \quad (9)$$

and $$\frac{\gamma}{\omega_{pe}} = \frac{A}{B} \quad (10)$$

with A being the same as in (5a) or (8a) while

$$B = \frac{\delta_H \eta_H}{\quad} \quad 1 + 3 \left| \frac{\alpha_H}{U\cos\theta\left(\frac{X}{Y} - \right.} \right.$$

$$4\delta_{O^+}\eta_{O^+}\frac{X}{Y}\left(\frac{U\cos\theta}{\alpha_{O^+}}\right)^4\left[1 - \frac{4}{3}\left(\frac{X}{Y}\frac{U\cos\theta}{\alpha_{O^+}}\right)\right.$$

$$\frac{\delta_{O^-}\eta_{O^-}}{\frac{X}{Y}} \quad 1 + 3\left(\frac{\alpha_{O^-}}{U\cos\theta}\right)^2 \frac{1}{\left(X\right)^2} \tag{11}$$

The other factors being the same we find that the hot O^+ ions aid the instability while the cold O^- ions tend to damp the wave.

3. Discussion

Although we have derived analytical expressions for the dispersion relation and growth rate, eqs. (3) and (4), (6) and (7), and (9) and (10) are sets of coupled equations which, as they stand, must be solved numerically. However, when $X \approx Y$, these sets of equations can be decoupled and considerable, interesting information obtained from them. We shall thus consider each case separately.

3.1. Case (i) Hot O^+ and O^- ions :

As stated above, we now consider the case of $X \approx Y$. We also assume that the growth/damping rate $\frac{\gamma}{\omega_{pe}} \ll 1$. Eq. (3) can thus be rewritten as

$$-\frac{\delta_H \eta_H}{P} \qquad \frac{3}{2}\frac{\delta_H \eta_H}{P}\left(\frac{\alpha_H}{U\cos\theta}\right)^2 = 0 \tag{12}$$

with a solution

$$(Y - X)^2 = \frac{\delta_H \eta_H Y^2}{2\left| Y^2 + 2\left(\frac{U\cos\theta}{\alpha_e}\right)^2 + 2\sum_{j=O^+,O^-} \eta_j\delta_j\left(\frac{U\cos\theta}{\alpha_j}\right)^2 \right|} \tag{13}$$

$$\left\{1 \pm \left[1 \cdot \frac{6\left| Y^2 + 2\left(\frac{U\cos\theta}{\alpha_e}\right)^2 + 2\sum_{j=O^+,O^-} \eta_j\delta_j\left(\frac{U\cos\theta}{\alpha_j}\right)^2\right.}{\delta_H\eta_H\left(\frac{U\cos\theta}{\alpha_H}\right)^2}\right.\right.$$

where

$$P = Y^2 + 2\left(\frac{U\cos\theta}{\alpha_e}\right)^2 + 2\sum_{j=O^+,O^-} \eta_j\delta_j\left(\frac{U\cos\theta}{\alpha_j}\right)^2. \tag{14}$$

If we now assume that Y^2 and $(U\cos\theta/\alpha_j)$ are very small compared to the oxygen terms in (14) (in contrast to the case discussed in [18] where $2(U\cos\theta/\alpha_e)^2$ was considered as the dominant term), (13) can be further simplified to

$$1 - \frac{X}{Y} = \frac{1}{2U\cos\theta\sum_{j=O^+,O^-}\left(\frac{n_j}{n_H}\right)^{\frac{1}{2}}\frac{1}{V_j}}$$

$$1 \pm \left[1 + 12\sum_{j=O^+,O^-}\frac{n_j}{n_H}\frac{T_H}{T_j}\right]^{\frac{1}{2}}\Bigg]^{\frac{1}{2}} \tag{15}$$

whence

$$\omega_r = k_\parallel U \cdot$$

$$_{j=O^+,O^-}\left(\frac{n_j}{n_H}\right)^{2}\frac{1}{V_j}$$

$$1 + \left[1 + 12\sum_{j=O^+,O^-}\frac{n_j}{n_H}\frac{T_H}{T_j}\right]^{\frac{1}{2}}$$

where

$$V_j^2 = \frac{T_j}{m_H}, \; j = O^+, O^- \text{ or}$$

As a check on (16) we note that it closely resembles equation (13) of [18]. As is evident from (16), the frequency of the ion acoustic wave is now dependent only on the number densities of the ions and their temperatures. As regards the expression for the growth/damping rate, eq. (4), the quantity A reduces to

$$A = \sqrt{\pi}x\left(\frac{U\cos\theta}{\alpha_e}\right)^3\left\{1 + \sum_{j=O^+,O^-}\left(\frac{\alpha_e}{\alpha_j}\right)^3\delta_j\eta_j\right. \tag{17a}$$

while B reduces to

$$B = \cdot\frac{\delta_H\eta_H}{\quad} \tag{17b}$$

Since from (15), $\left(1 - \frac{X}{Y}\right)$ depends very sensitively on the oxygen-ion densities and temperatures, the growth/damping rate is also dependent on these factors.

3.2. Case (ii) Cold O^+ and O^- ions :

We had in the above section, discussed limiting cases of the dispersion relation and growth/damping rate, when the O^+ and O^- ions were hot, under conditions of $X \approx Y$ and $\frac{\gamma}{\omega_{pe}} \ll 1$. We now carry out a similar discussion when the O^+ and O^- ions are cold.

Expression (6) can be rewritten as

$$\frac{\delta_H\eta_H}{P}\left(1 - \frac{X}{Y}\right)^2 - \frac{3}{2}\frac{\delta_H\eta_H}{P}\left(\frac{\alpha_H}{U\cos\theta}\right)^2 = 0 \tag{18}$$

which has a solution.

$$(Y - X)^2 = \frac{\delta_H\eta_H Y^2}{2\left| Y^2 + 2\left(\frac{U\cos\theta}{\alpha_e}\right)^2 - \sum_{j=O^+,O^-}\delta_j\eta_j\right.}$$

$$1 \pm \; 1 + \frac{6\left\lfloor Y^2 + 2\left(\frac{U\cos\theta}{\alpha_e}\right)\right\rfloor}{\delta_H\eta_H}\frac{-\sum_{j=O^+,O^-}\delta_j\eta_j}{\delta_H\eta_H\left(\frac{U\cos\theta}{\alpha_H}\right)^2} \tag{19}$$

where

$$p = Y^2 + 2\left(\frac{U\cos\theta}{\alpha_e}\right) - \Sigma\delta_j\eta_j; j = O^+, O^- .$$

From (19), we see that the dispersion characteristics sensitively depend on the densities of the O^+ and O^- ions. Since the electrons are assumed hot, they can easily satisfy the conditions

$$(U\cos\theta)^2 = \frac{1}{2}\sum_{j=O^+,O^-} \delta_j\eta_j\alpha_e .$$

Under such conditions, the solution (19) reduces to

$$(Y-X)^2 = \frac{\delta_H\eta_H}{2}\left[1 \pm \sqrt{1 + \frac{6Y^2}{\delta_H\eta_H\left(\frac{U\cos\theta}{\alpha_H}\right)^2}}\right] \quad (2$$

or in other words, the dispersion characteristics of the ion-acoustic wave now depend only on the parametres of the hydrogen ion.

Considering expression (7) for the growth/damping rate, we have

$$A = \sqrt{\pi}x\left[\frac{U\cos\theta}{\alpha_e}\right]^3$$

while the expression for B is

$$B = \frac{\delta_H\eta_H}{\left(1-\frac{X}{Y}\right)^3} + \Sigma_{j=O^+,O^-}\delta_j\eta_j$$

For this case, the growth/damping rate depends only on the O^+ and O^- densities and not on their temperatures.

Finally, we consider the case where the O^+ ions are hot and the O^- are cold. Expression (9) for Y^2, can be reduced to the bi-quadratic equation similar to (12) and (15) with the parameter p being given by

$$p = Y^2 + 2\left(\frac{U\cos\theta}{\alpha_e}\right)^2 + 2\delta_{O^+}\eta_{O^+}\left(\frac{U\cos\theta}{\alpha_{O^+}}\right) - \delta_{O^-}\eta_{O^-} \quad (21)$$

Again the dispersion characteristics of the ion acoustic wave are influenced by the densities of the O^+ and O^- ions. Also the expression for A and B are now

$$A = \sqrt{\pi}x\left(\frac{U\cos\theta}{\alpha_e}\right)^3\left\{1 + \left(\frac{\alpha_e}{\alpha_{O^+}}\right)\delta_{O^+}\eta_{O^+}\right\}$$

and

$$B = \frac{\delta_H\eta_H}{\left(1-\frac{X}{Y}\right)^3} - \delta_{O^-}\eta_{O^-}$$

4. Results

We now consider our dispersion relations and expressions for the growth/damping rate for typical parameters observed near the coma of comet Halley by the Giotto spacecraft. Negative ions, in three broad mass peaks at 7–19, 22–65 and 85–110 amu with densities reaching $\eta \geq 1$, 5×10^{-2} and 4×10^{-2} cm^{-3} were observed. Their energies ranged between 0.03 eV to 3.0 KeV with a background of 1.0 KeV. Of the many ionic species, O^- was unambiguously identified [13]. We therefore, assume that the oxygen ions have temperatures equivalent to 3.0 KeV (when they are hot) and 0.03 eV (when they are cold). A temperature equivalent of 1.0 KeV was assigned to the hydrogen ions. The density of the oxygen ions was assumed to be 1.0, and that of hydrogen, 3.0. These assignments of temperatures and densities are in good agreement with the observed values. The electrons were assumed to have a temperature of 2×10^5 °K while the hydrogen ions were assigned a drift velocity of 4.0×10^7 cm s^{-1} [11,12].

Eqs. (3) and (4), (6) and (7) and (9) and (10) are sets of nonlinear equations that have to be solved by numerical methods. They were solved iteratively and convergence was achieved after a few hundred iterations; we present below the salient results. The values of the thermal velocity of the electrons and the drift velocity of the hydrogen ions were held a constant at the above values throughout our computations.

Figure 1 is a plot of the growth rate *versus X/Y* for $\eta_H = 3.0$; $\eta_{O^+} = \eta_{O^-} = 1.0$ with the thermal energies of both O^+ and O^- ions equal to 3.0 KeV as a function of cos θ (= 0.6, 0.7, 0.8). The growth rate is low for cos θ = 0.6; the wave even damps beyond a X/Y of 0.375. However, for cos θ = 0.7 and 0.8, the wave is unstable over the entire region of X/Y studied and the growth rate increases with increasing cos θ, or in other words, the wave is most unstable for parallel propagation. This is a well known characteristic of the ion-acoustic wave.

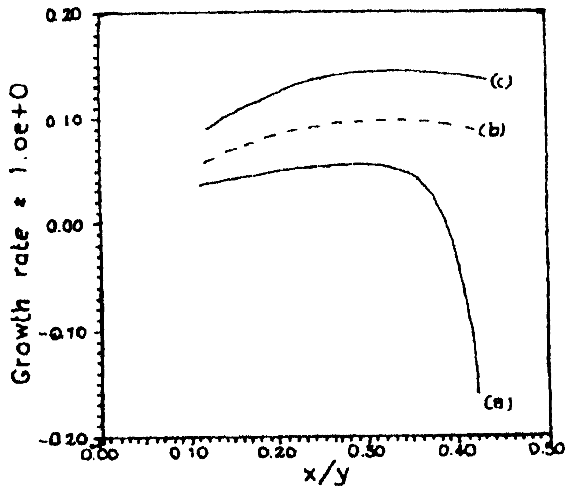

Figure 1. Plot of γ/ω_{pe} *versus X/Y* for $\eta_H = 3.0$, $\eta_{O^+} = \eta_{O^-} = 1$ for $\alpha_{O^+} = \alpha_{O^-} = 3.0$ KeV as a function of cos θ (= 0.6, curve (a), = 0.7, curve (b) and = 0.8, curve (c)).

In order to understand the influence of heavy ions on the growth/damping rate of the ion acoustic wave we plot, in Figure 2, $\frac{\gamma}{\omega_{pe}}$ *versus X/Y* for cos θ = 0.8 and $\eta_H = 3.0$ as a function of η_{O^+} or η_{O^-}. The growth rate is found to be the lowest when the heavy ions are absent, curve (a). Compared to the growth rate in a single ion plasma, the growth rate increases in the presence of the heavier ions ($\eta_H = 3.0$, η_{O^+} or $\eta_{O^-} = 1$; curve (b)). Finally wave growth is largest when both the heavy ions are present ($\eta_H = 3.0$, $\eta_{O^+} = \eta_{O^-} = 1$, curve (c)). The heavy, hot ions thus have a strong influence in increasing the instability of the ion-acoustic wave.

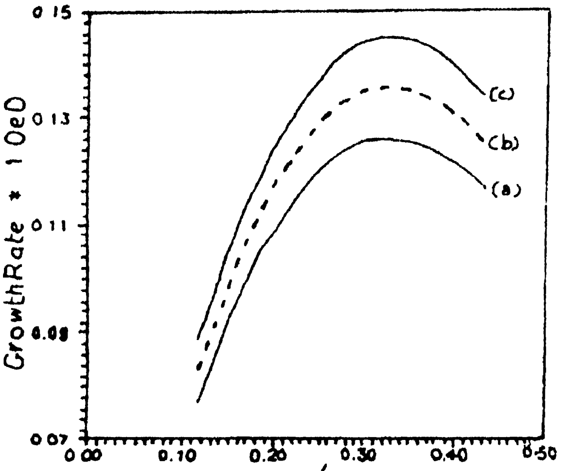

Figure 2. Plot of γ / ω_{pe} *versus* X/Y for $\eta_H = 3.0$, $\cos\theta = 0.8$ as a function of $\eta_{O^+} = \eta_{O^-}$ with $\alpha_{O^+} = \alpha_{O^-} = 3.0$ KeV. Curve (a) corresponds to a single ion plasma, curve (b) to a two-ion plasma ($\eta_H = 3.0$, η_{O^+} or $\eta_{O^-} = 1$) and curve (c) for a three ion plasma ($\eta_H = 3.0$, $\eta_{O^+} = 1.0$ and $\eta_{O^-} = 1.0$)

The other parameter that could influence the stability of the wave is the density of the lighter ion, hydrogen. Figure 3 thus depicts the variation of $\frac{\gamma}{\omega_{pe}}$ *versus* X/Y for $\cos\theta = 0.8$, $\eta_{O^+} = \eta_{O^-} = 1.0$ as a function of η_H. Curve (a) depicts the variation in the growth rate for $\eta_H = 2.5$, curve (b) for $\eta_H = 3.0$ and curve (c) for $\eta_H = 3.5$. The curves indicate a slight decrease in the growth rate with increasing densities of the lighter ion, hydrogen.

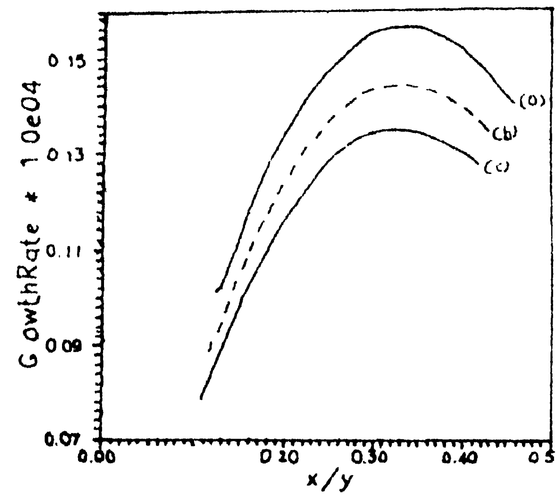

Figure 3. Plot of γ / ω_{pe} *versus* X/Y for $\eta_{O^+} = \eta_{O^-} = 1.0$ with $\alpha_{O^+} = \alpha_{O^-} = 3.0$ KeV and $\cos\theta = 0.8$ as a function of η_H. Curves (a), (b) and (c) correspond respectively to $\eta_H = 2.5$, 3.0 and 3.5.

We next consider the influence of the cold, heavier ions on the stability of the ion-acoustic wave. As stated above both the O^+ and O^- ions now have a temperature equivalent of 0.03 eV; the temperature of both electrons and hydrogen-ions remaining unchanged.

Figure 4 thus depicts the variation in the growth rate *versus* X/Y as a function of $\cos\theta$(= 0.6, curve (a); = 0.7, curve (b); = 0.8 curve (c)) for $\eta_H = 3.0$ and $\eta_{O^+} = \eta_{O^-} = 1.0$

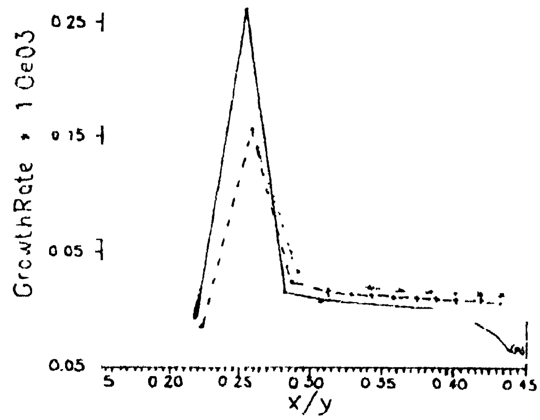

Figure 4. Plot of γ / ω_{pe} *versus* X/Y for $\eta_H = 3.0$, $\eta_{O^+} = \eta_{O^-} = 1.0$ with $\alpha_{O^+} = \alpha_{O^-} = 0.03$ eV as a function of $\cos\theta$ (= 0.6, curve (a), = 0.7, curve (b) and = 0.8, curve (c))

A comparison of Figures 1 and 4 reveals several interesting features : (i) the growth rate now is nearly an order of magnitude larger when compared to that in Figure 1, (ii) while, in Figure 1, the wave was damped at the higher end of X/Y for $\cos\theta = 0.6$, the wave is now damped at both the higher and lower ends of X/Y. The damping could be due to the fact that the heavier ions are now cold and the consequent increased Landau damping, and (iii) the peak value in the growth rate increases with decreasing $\cos\theta$ with a gradual shift towards lower X/Y. However in agreement with Figure 1 the growth rate, in general, increases with increasing $\cos\theta$.

Similar to Figure 2, Figure 5 is intended to bring out the influence of the heavy ions on the stability of the ion acoustic

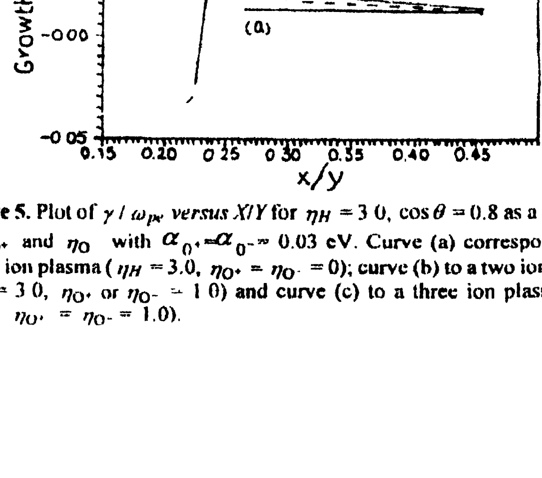

Figure 5. Plot of γ / ω_{pe} *versus* X/Y for $\eta_H = 3.0$, $\cos\theta = 0.8$ as a function of η_{O^+} and η_{O^-} with $\alpha_{O^+} = \alpha_{O^-} = 0.03$ eV. Curve (a) corresponds to a single ion plasma ($\eta_H = 3.0$, $\eta_{O^+} = \eta_{O^-} = 0$); curve (b) to a two ion plasma ($\eta_H = 3.0$, η_{O^+} or $\eta_{O^-} = 1.0$) and curve (c) to a three ion plasma ($\eta_H = 3.0$, $\eta_{O^+} = \eta_{O^-} = 1.0$).

wave. Thus it depicts the variation in $\frac{\gamma}{\omega_{pe}}$ *versus X/Y* for cos θ = 0.8 and η_H = 3.0 as a function of η_{O^+} and η_{O^-}. It is seen that in a single ion plasma (η_{O^+} = η_{O^-} = 0, curve (a)) the growth rate is low and approximately a constant. It increases slightly when either O^+ or O^- ions are added (η_{O^+} or η_{O^-} = 1 curve, (b)); the peak in the growth rate increases tremendously when both types of heavy ions are included (η_{O^+} = η_{O^-} = 1, curve, (c)). However, in contrast to Figure 2, the growth rate is almost independent of the heavier ion densities for higher values of *X/Y*.

A plot similar to Figure 3 was made to study the variation in the growth/damping rate with the hydrogen ion density. Figure 6 thus depicts a plot of $\frac{\gamma}{\omega_{pe}}$ *versus X/Y* as a function of η_H (= 2.5, indicated by curve (a) and 3.0, curve (b)). It is seen that the peak in the growth rate, depicted in the Figures 4 and 5, shows a sharp decrease with decreasing η_H. However, similar to Figure 3, the growth rate shows a slight decrease with increasing η_H after the peak value.

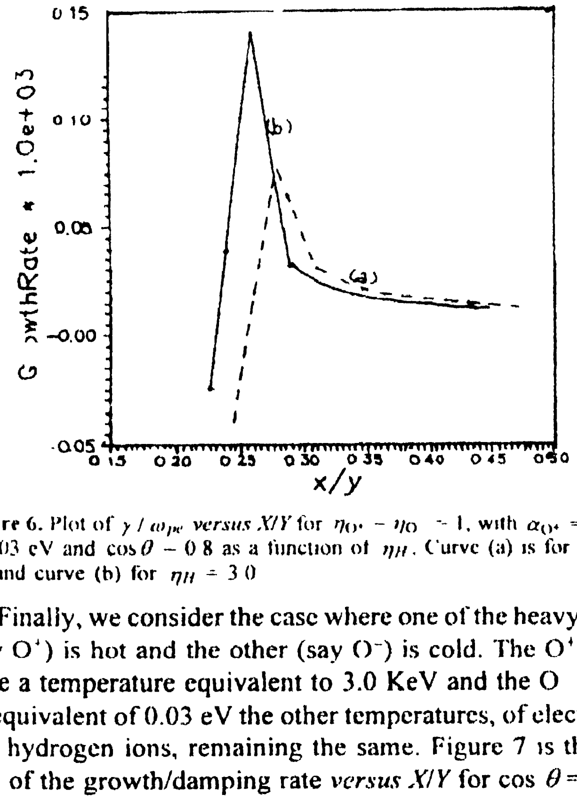

Figure 6. Plot of γ / ω_{pe} *versus X/Y* for η_{O^+} = η_{O^-} = 1, with α_{O^+} = α_{O^-} = 0.03 eV and cos θ = 0.8 as a function of η_H. Curve (a) is for η_H = 2.5 and curve (b) for η_H = 3.0

Finally, we consider the case where one of the heavy ions (say O^+) is hot and the other (say O^-) is cold. The O^+ ions have a temperature equivalent to 3.0 KeV and the O^- ions, an equivalent of 0.03 eV the other temperatures, of electrons and hydrogen ions, remaining the same. Figure 7 is thus a plot of the growth/damping rate *versus X/Y* for cos θ = 0.6, 0.7 and 0.8 with η_H = 3.0 and η_{O^+} = η_{O^-} = 1.0. For cos θ = 0.6 (curve (a)) the wave damps for both low and high *X/Y*; for other values of cos θ the wave is unstable at higher *X/Y*. The growth rate is again found to increase with increasing cos θ (= 0.7, curve (b); = 0.8, curve (c)).

The influence of the heavy ions on the growth/damping rate was found to be similar to Figures 2 and 5; the hot ions (η_{O^+} = 1, η_{O^-} = 0) tending to aid the instability while the cold ions (η_{O^+} = 0, η_{O^-} = 1.0) tending to damp the wave.

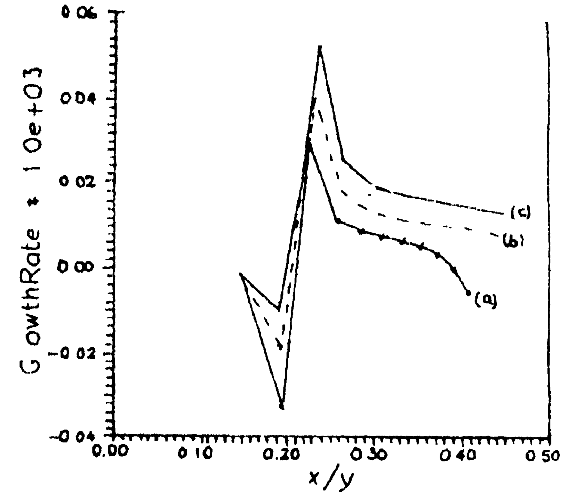

Figure 7. Plot of γ / ω_{pe} *versus X/Y* for η_{O^+} = 1.0 (α_{O^+} = 3.0 KeV), η_{O^-} = 1.0, α_{O^-} = 0.03 eV) with η_H = 3.0 as a function of cos θ (= 0.6, curve (a), = 0.7, curve (b) and = 0.8, curve (c))

As mentioned in Section 1, ion acoustic waves in the frequency range of a few hundred KHz were observed in the plasma environments of comets Halley [16] and Giacobini-Ziner [17]. Our calculations, for typical values of parameters observed in the coma of comet Halley, indicate that the ion-acoustic waves can be driven unstable by the heavier ions when they are hot. While these heavy, hot ions may not be the only agent that drives the ion-acoustic wave unstable, they can certainly add to the instability of the observed ion-acoustic waves.

5. Conclusions

We have, in this paper, studied the stability of the ion-acoustic wave in a multi-ion plasma comprising of drifting hydrogen and heavy ions O^+ and O^-. Expressions for the dispersion relations and the growth/damping rates, for oblique propagation, were derived for various temperature conditions of the heavier ion species. The wave is most unstable for parallel propagation with the heavier ions tending to aid the growth when they are hot and damping the instability when they are cold.

The hot, heavier oxygen ions can thus enhance the instability of the ion-acoustic waves which have been observed in the plasma environments of comets Halley and Giacobini-Ziner.

Acknowledgment

We thank the referee for the very useful comments which have greatly improved the scientific content of this paper.

References

[1] D D Sentman, J P Edminston and L A Frank *J Geophys Res* **86** 7487 (1981)

[2] C S Wu and R C Davidson *J Geophys. Res.* **77** 5399 (1972)

[3] D Winske, C S Wu, Y Y Li, Z Z Mov and S Y Guo *J Geophys Res* **90** 2713 (1985)

[4] M L Goldstein and H K Wong *J. Geophys Res.* **92** 4695 (1987)

[5] S P Gary, C W Smith, M A Lee, M L Goldstc and D W Forslund *Phys. Fluids* **27** 1852 (1984)

[6] O P Sharma and V L Patel *J Geophys. Res* **91** 1529 (1986)

[7] S P Gary and D Winske *J Geophys Res* **91** 13, 699 (1986)

[8] A L Brinca and B T Tsurutani *J Geophys Res* **93** 243 (1988)

[9] B T Tsurutani, A L Brinca, E J Smith, R M Thorne, F L Scarf, J T Gosling and F M Jpavich *Astron Astrophys.* **187** 92 (1987)

[10] A L Brinca and B T Tsurutani *Geophys Res Lett* **14** 495 (1987)

[11] A L Brinca and B T Tsurutani *Astron. Astrophys* **187** 311 (1987)

[12] A L Brinca and B T Tsurutani *J. Geophys. Res.* **93** 48 (1988)

[13] P H Chaizy, H Reme, J A Sauvaud, C d'Uston, R P Lin, D E Larson, D L Mitchell, K A Anderson, C W Carlson, A Korth and D A Mendis *Nature* **349** 393 (1991)

[14] B Song, N D' Angelo and R L Merlin *Phys. Fluids* **B3** 284 (1991)

[15] V W Chow and M Rosenberg *Phys. Plasmas* **3** 1202 (1996)

[16] H Oya, A Moioka, W Miyake, E J Smith and B T Tsurutani *Nature* **321** 307 (1986)

[17] F L Scarf, F V Coroniti, C F Kennel and D A Gurnett *Nature* 332 377 (1986)

[18] L Gomberroff and P Vega *Plasma. Phys and Controll Fusion* 32 737 (1990)

[19] B D Fried and S D Conte *The Plasma Dispersion Function* (San Diego, California Academic) (1961)